Selection Of Mixed Sampling Plan With CSP-1 (C=2) Plan As Attribute Plan Indexed Through MAPD AND MAAOQ

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Abstract - In this paper a procedure for the construction and selection of the independent mixed sampling plan using MAPD and MAAOQ as quality standards with Continuous Sampling plan of the type CSP-1 (c=2) as attribute plan is presented. Tables are constructed for the selection of parameters of the plan when MAPD and MAAOQ are given. Practical applications of the sampling plan are also discussed with suitable example.

Key words and Phrases: Maximum allowable percent defective, maximum allowable average outgoing quality, Operating characteristic.
AMS (2000) Subject classification Number: Primary: 62P30, Secondary: 62D05.

1. Introduction

A variety of plans and procedures have been developed for special sampling situation involving both measurements and attributes. Each is tailored to do a specific job under prescribed circumstances. They range from a simplified variables approach to a more technically complicated combination of variables and attribute sampling called mixed sampling plans.

Mixed sampling plans are of two types, namely independent and dependent plans. Independent mixed plans do not incorporate first sample results in the assessment of the second sample. Dependent mixed plans combine the results of the first and second samples in making a decision if a second sample is necessary.

Mixed Sampling Plan (MSP) was first developed by Schilling (1967) for the case of single sided specifications, standard deviation known by assuming an underlying normal distribution for measurements. Dodge (1943) provided the concept of continuous sampling inspection and introduced the first continuous sampling plan. Dodge (1947) outlined several sampling plans for continuous production, originally referred to as the random – order and later designated as CSP-1 plan by Dodge and Torrey (1951). The desirability of developing a set of sampling plans indexed with p_* has been explained by, Soundararajan (1975), Kandasamy (1993) studied in designing of various types of continuous sampling plans. Suresh and Ramkumar (1996)discussed about the use of MAAOQ for the selection of sampling plans. Radhakrishnan (2002) constructed various continuous sampling plans indexed through MAAOQ and mentioned its advantage over AOQL. Devaarul (2003), Sampath Kumar (2007), Radhakrishnan and Sampath Kumar (2006, 2007, 2009), Radhakrishnan et.al (2010) have made contributions to mixed sampling plans for independent case. Radhakrishnan et.al and (2009) studied mixed sampling plan for dependent case.

In this paper, using the operating procedure of mixed sampling plan (independent case) with CSP-1 (c=2) as attribute plan, tables are constructed for the mixed sampling plan indexed through MAPD and MAAOQ. Suitable suggestions are also provided for the future.

2. Glossary of Symbols

The symbols used in this paper are as follows

- P : submitted quality of lot or process
- p_* : maximum allowable percent defective (MAPD)
- β_j : probability of acceptance for lot quality 'p_j'
- β'_j : probability of acceptance assigned to first stage for percent defective 'p_j'
- β''_j : probability of acceptance assigned to second stage for percent defective 'p_j'
- k : variable factor such that a lot is accepted if $\overline{X} \geq A = L + k\sigma$
- f : the rate of inspection(=1/n)
- i : number of consecutive units are found conforming
- n_1 : sample size for the variable sampling plan
- n_2 : sample size for the attribute sampling plan = (1/f) units

3. Formulation of Mixed Sampling Plan with CSP-1 (c=2) as Attribute Plan

The development of mixed sampling plans and the subsequent discussions are limited only to the lower specification limit 'L'. By symmetry a parallel discussion can be used for upper specification limits also It is suggested that the mixed sampling plan with CSP–1(c=2) in the case of single sided specification (L), standard deviation (σ) known can be formulated by the parameters i, n_1, n_2 and k.

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Mixed sampling procedure suggested by Schilling (1967) is slightly modified and presented in this paper and this procedure is to be adopted separately for each time period fixed by the manufacturer. By giving the values for the parameters an independent plan for single sided specification, σ known would be carried out as follows:

- Determine the parameters with reference to ASN and OC curves
- Take a random sample of size n_1 from the lot assumed to be large during the time period't' (may be an hour / a shift / a day / a week …). [This is the modification suggested in this paper over Schilling (1967)]
- If a sample average $\overline{X} \geq A = L + k\sigma$, accept the lot
- If the sample average $\overline{X} < A = L + k\sigma$, apply the operating procedure of CSP-1 (c=2)

Operating Procedure of CSP-1(c=2) plan

Step 1: At the outset, inspect 100% of the units consecutively in the order of production until i successive conforming units are found.

Step 2: When i units in succession are found conforming discontinue 100% inspection and inspect f (=1/n) units until total of (c+ 1) sampled units are found nonconforming.

Step 3: When the number of nonconforming sampled units reaches (c+1), revert to 100% inspection as in step 1.

Step 4: Correct or replace all nonconforming units found with conforming units.

Step 5: At the end of the time t switch back to variable sampling plan for the units produced.

4. Construction and Selection of Mixed Sampling Plan having CSP-1 (c=2) as attribute plan indexed through MAPD and MAAOQ

Maximum allowable percent defective (MAPD) is the quality level that corresponds to the point of inflection of the OC curve. It is the quality level at which the second order derivative of the OC function $P_a(p)$ with respect to p is zero ans the third order not equal to zero. When some specific value for a characteristic or group of characteristics is designated, the continuous sampling plan will have a tendency to accept product during periods of sampling if the submitted quality is upto MAPD and if the submitted quality is beyond MAPD, the sampling plan will have a tendency to submit the product for screening. The inflection point (p_*) is obtained by using $d^2P_a(p)/dp^2 = 0$ and $d^3P_a(p)/dp^3 \neq 0$.

Maximum allowable average outgoing quality (MAAOQ) of a sampling plan is designated as the Average Outgoing Quality (AOQ) at the MAPD. AOQ = p. $P_a(p)$. and MAAOQ = AOQ at p = p_*. Schilling (1967) has given the procedure for constructing the mixed sampling plan when a point on OC curve and n_1 are known with CSP-1(c=2) as attribute plan for a specific (p_*, β_*), n_1, n_2 and i

- Assume that the mixed sampling plan is independent
- Split the probability of acceptance (β_j) determining the probability of acceptance that will be assigned to the first stage. Let it be β'_j
- Decide the sample size n_1 (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as $A = L + k\sigma = L + [z(p_j) + \{z(\beta'_j)/\sqrt{n_1}\}]\sigma$, Where z(t) is the standard normal variate corresponding to 't' such that $t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$.
- Determine the sample average $\overline{X}$. If a sample average $\overline{X} < A = L + K\sigma$, take a second stage sample of size 'n_2' using attribute sampling plan.
- Now determine β''_j the probability of acceptance assigned to the attributes plan associated with the second stage as $\beta''_j = \frac{\beta_j - \beta'_j}{1 - \beta'_j}$
- Determine the values of n_2 and i from Pa(p)= β''_j for p = p_j.

Using the above procedure tables are constructed to facilitate easy selection of mixed sampling plan with CSP-1(c=2) as attribute plan indexed through MAPD and MAAOQ.

Construction of Tables

According to Stephens (1979), the OC function of the CSP-1(c=2) plan is given by $P_a(p) = 3q^i/[f+(3-f)q^i]$, where p is the fraction of incoming lots that are not acceptable and q=1-p and f=1/n. The values of i and n are calculated for different possible combinations of MAPD and MAAOQ for $\beta_*' = 0.20$ using Visual Basic Program and presented in Table 1.

Selection of the Plan

For the specified values of MAPD and MAAOQ one can find the ratio MAPD / MAAOQ and select the nearest value of the ratio MAPD/MAAOQ in Table 1 and corresponding to the given value of MAPD, the values of i and n_2 are obtained from Table 1.

Example : 1

Given MAPD = 0.0015 and MAAOQ = 0.000599. Compute the ratio MAPD/MAAOQ = 2.5042 and select the nearest value of the ratio from the Table 1 as 2.5000. The values of i and n_2 corresponding to the ratio 2.5000 and MAPD = 0.0015 are i = 2393, n_2 = 8 and f = 1/n_2 = 1/8 = 0.125. Thus i = 2393, f = 0.08 are the parameters selected for the mixed sampling plan having CSP-1(C=2) plan as attribute plan for a specified MAPD = 0.0015 and MAAOQ = 0.000599.

Practical Application of Mixed Sampling Plan with CSP-1(c=2) plan as attribute Plan

The biscuits are expected to contain sugar, carbohydrates, fat, etc., in a specified proportion. Inspection of such items means testing one or more characteristics of the product. Here mixed sampling plan with CSP-1(c=2) as attribute plan can be used as a basis for acceptance or rejection of such items. The characteristic to be inspected is the "Carbohydrates " of the item for which there is a specified lower limit of 78.1g specified by the producer with a known standard deviation (σ) of 0.2g.

In this example, L = 78.1g, σ = 0.2g and k = 1.5.

$$A = L + K\sigma = 78.1 + (1.5)(0.2) = 78.4g.$$

Now, by applying the variable inspection first, take a random sample of size n_1 = 10 and find the sample average $\bar{X}$ of the characteristic. If $\bar{X} \geq 78.4$g accept the units otherwise apply the attribute inspection. Under attributes inspection, by taking CSP-1(c=2) as attribute plan, for the specified MAPD = 0.0015 (15 non-conformities out of 10000 units) and MAAOQ = 0.000599 (599 non-conformities out of 10, 00,000 units) the value of i = 2393 and n_2 = 8.

- Apply 100% inspection to the submitted units and such inspection is continued until i = 2393 units in succession are found to be acceptable.
- When i = 2393 consecutive units are found acceptable, then 100% inspection is discontinued and an inspection at the rate of f = 1/n_2 = 1/8 = 0.125 units until total of (c+1) or three sampled units are found nonconforming.
- During the skipping inspection, when the number of nonconforming sampled units reaches (c+1) or three, apply 100% inspection from the next unit and inform the management for improving the quality of the product.
- At the end of the time t switch back to variable sampling plan for the units produced.

The OC curve for the plan i = 2393 and n_2 = 8 is shown in Figure 1:

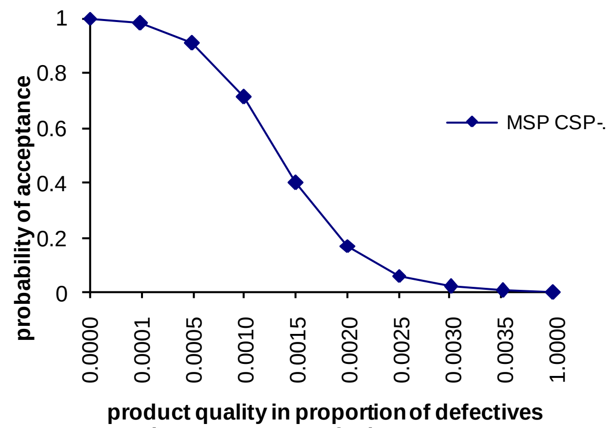

Figure 1: OC curve for i = 2393, n_2 = 8

6. Conclusion

In this paper a procedure for the construction and selection of independent mixed sampling plan having CSP-1(c=2) as attribute plan is presented. A table is also provided for the easy selection of the plans when MAPD and MAAOQ are known. Practical application for the sampling plan is also discussed. If the floor engineers know the levels of MAPD and MAAOQ, they can have their sampling plans on the floor itself by referring to the tables. This provides the flexibility to the floor engineers in deciding their sampling plans. Various plans can also be constructed to make the system user friendly by changing the first stage probabilities ($\beta^{*\prime}$). Similar plans can also be constructed for dependent mixed sampling plan suggested by Radhakrishnan et.al (2009).

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Table 1: **Various characteristics of Mixed Sampling plan with CSP-1 (C=2) plan as attribute plan indexed by MAPD and MAAOQ when $\beta_*' = 0.20$.**

MAPD/ MAAOQ	MAPD in Percent																			
	0.15		0.25		0.4		0.65		1.0		1.5		2.5		4.0		6.5		10.0	
	I	n_2	i	n_2	I	n_2	i	n_2	i	n_2	i	n_2	i	n_2	i	n_2	i	n_2	I	n_2
2.6490	3749	56	2324	67	1342	43	814	40	542	46	320	25	172	15	97	10	59	10	40	13
2.6316	3139	23	1883	23	1242	30	694	19	427	15	306	21	154	10	96	10	41	3	43	18
2.6059	2981	18	1788	18	1072	15	678	17	414	13	280	14	146	8	93	9	60	11	31	5
2.5806	2940	17	1739	16	1071	15	668	16	405	12	275	13	150	9	90	8	57	9	29	4
2.5641	2684	12	1610	12	1025	13	591	10	393	11	261	11	153	10	83	6	62	13	29	4
2.5477	2634	11	1580	11	1008	12	576	9	374	9	262	11	144	8	83	6	51	6	37	10
2.5157	2496	9	1497	9	984	11	557	8	362	8	241	8	144	8	83	6	55	8	36	9
2.5000	2393	8	1435	8	925	9	508	6	358	8	238	8	142	8	72	4	34	2	36	9
2.4615	2393	8	1324	6	896	8	508	6	330	6	220	6	116	4	77	5	47	5	36	9
2.4539	2207	6	1324	6	896	8	508	6	330	6	220	6	116	4	77	5	47	5	26	3
2.4169	2184	6	1310	6	888	8	476	5	309	5	192	4	104	3	65	3	47	5	25	3
2.4024	2064	5	1238	5	773	5	476	5	309	5	191	4	123	5	65	3	47	5	25	3
2.3952	2057	5	1149	4	720	4	441	4	261	3	174	3	123	5	65	3	47	5	25	3

2.3599	1721	3	1032	3	645	3	397	3	221	2	171	3	121	5	64	3	46	5	25	3
2.3121	1699	3	873	2	545	2	335	2	218	2	145	2	120	5	63	3	46	5	25	3